

Calculation of nuclear matter in the presence of strong magnetic field using LOCV technique

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Abstract

In the present work, we are interested in the properties of nuclear matter at zero temperature in the presence of strong magnetic fields using the lowest order constraint variational (LOCV) method employing AV_{18} nuclear potential. Our results indicate that in the absence of a magnetic field, the energy per particle is a symmetric function of the spin polarization parameter. This shows that for the nuclear matter, the spontaneous phase transition to a ferromagnetic state does not occur. However, we have found that for the magnetic fields $B \gtrsim 10^{18} \text{ G}$, the symmetry of energy is broken and the energy has a minimum at a positive value of the spin polarization parameter. We have also found that the effect of magnetic field on the value of energy is more significant at the low densities. Our calculations show that at lower densities, the spin polarization parameter is more sensitive to the magnetic field.

Keywords: Nuclear matter, magnetic field, magnetic properties

I. INTRODUCTION

New discoveries related to the magnetic field of a neutron star have led to the theoretical researches on the magnetic properties of the dense matter. The magnetic field of a neutron star may originate from the compression of magnetic flux inherited from the progenitor star [1]. Using this idea, Woltjer has predicted a magnetic field strength of order 10^{15} G for neutron stars [2]. Moreover, general relativity predicts the allowed maximum value of neutron star magnetic field to be about $10^{18} - 10^{20}$ G [3]. By comparing with the observational data, Yuan et al. obtained a magnetic field strength of order 10^{19} G for neutron stars [4]. In the core of inhomogeneous gravitationally bound dense magnetars, the magnetic field strength can be as large as 10^{20} G [5]. In addition, considering the formation of a quark core in the interior of a magnetar, the maximum field reaches up to about 10^{20} G [5, 6].

These very intense magnetic fields have significant effects on the dense nuclear matter. Some studies have investigated the properties of neutron star matter and nuclear matter [7–13] in strong magnetic fields. The instabilities of nuclear matter was studied considering the relativistic nuclear models [8]. They found that the presence of the magnetic field will generally increase the instability region. The effects of strong magnetic fields on nuclear matter were studied in the framework of the relativistic mean field models FSU-Gold by including the anomalous magnetic moments of the nucleons [10]. They concluded that at low densities, by increasing the magnetic field, the energy per particle turns out to be increasing lower and a softening of the equation of state appears. However, they showed that at high densities, while the softening of the EOS will be gradually overwhelmed by stiffening resulting from the anomalous magnetic moments effect, the energies are slightly reduced by a strong magnetic field. Employing two relativistic field-theoretical models for nuclear matter, the Walecka model and an extended linear sigma model, it has been found that the creation of nuclear matter in a sufficiently strong magnetic field becomes energetically more costly due to the heaviness of magnetized nucleons [11]. Using the semi-empirical mass formula of Green based on the liquid drop nuclear model, the onset of the neutron drip in high-density matter in the presence of a magnetic field has been investigated [12]. It has been found that for systems having only protons and electrons, in the presence of a magnetic

field $B \gtrsim 10^{15} G$, neutronization occurs at a density that is at least an order of magnitude higher compared to that in a nonmagnetic system. The effect of a strong magnetic field on the proton spin polarization and magnetic susceptibility of asymmetric nuclear matter has been studied within a relativistic mean-field approach [13].

In our previous works, we have studied the spin polarized neutron matter [14], symmetric nuclear matter [15], asymmetric nuclear matter [16], and neutron star matter [16] at zero temperature using the lowest order constraint variational (LOCV) method with the realistic strong interaction in the absence of magnetic field. We have also investigated the thermodynamic properties of these systems at finite temperature with no magnetic field [17–19]. Furthermore, we have calculated the properties of spin polarized neutron matter in the presence of strong magnetic fields at zero [20] and finite [21] temperatures. Very recently, we have investigated the temperature and density dependence of asymmetry energy of nuclear matter [22]. In the present work, we intend to extract the properties of nuclear matter in strong magnetic fields by LOCV method using AV_{18} potential.

II. LOCV FORMALISM FOR NUCLEAR MATTER

We study a system consists of A nucleons where $A^{(+)}$ nucleons are spin-up and $A^{(-)}$ nucleons are spin-down with the spin polarization parameter, ξ , as

$$\xi = \frac{A^{(+)} - A^{(-)}}{A^{(+)} + A^{(-)}}. \quad (1)$$

For calculation of nuclear matter properties, we apply LOCV approach. In this method, we consider up to the two-body cluster energy [28],

$$E = E_1 + E_2. \quad (2)$$

In above equation, E_1 is the total one-body energy of spin-up and spin-down nucleons,

$$E_1 = E_1^{(+)} + E_1^{(-)}. \quad (3)$$

and E_2 is the two-body energy contribution,

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij | \nu(12) | ij - ji \rangle, \quad (4)$$

in which $\nu(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12)$ is the two-body effective potential. Here, $V(12)$ is the two-body nuclear potential and $f(12)$ is the two-body correlation

function. To calculate the interaction energy of the nuclear matter using LOCV formalism, the two-body correlation function, $f(12)$, is considered as follows [25],

$$f(12) = \sum_{k=1}^3 f^{(k)}(r_{12}) P_{12}^{(k)}, \quad (5)$$

where

$$P_{12}^{(k=1-3)} = (\frac{1}{4} - \frac{1}{4}\sigma_1 \cdot \sigma_2), (\frac{1}{2} + \frac{1}{6}\sigma_1 \cdot \sigma_2 + \frac{1}{6}S_{12}), (\frac{1}{4} + \frac{1}{12}\sigma_1 \cdot \sigma_2 - \frac{1}{6}S_{12}). \quad (6)$$

In the above equation S_{12} and σ_1 and σ_2 are the tensor and Pauli operators, respectively. Using the above two-body correlation function and the AV_{18} two-body potential [26], we find the following equation for the two-body energy:

$$\begin{aligned} E_2 = & \frac{2}{\pi^4 \rho} \left(\frac{\hbar^2}{2m} \right) \sum_{JLTSS_z} \frac{(2J+1)(2T+1)}{2(2S+1)} [1 - (-1)^{L+S+T}] \left| \left\langle \frac{1}{2}\sigma_{z1} \frac{1}{2}\sigma_{z2} \mid SS_z \right\rangle \right|^2 \int dr \\ & \left\{ \left[f_\alpha^{(1)'}{}^2 a_\alpha^{(1)2}(k_F r) + \frac{2m}{\hbar^2} (\{V_c - 3V_\sigma + (V_\tau - 3V_{\sigma\tau})(4T-3) - (V_T - 3V_{\sigma T})(4T)\} \right. \right. \\ & \times a_\alpha^{(1)2}(k_F r) + [V_{l2} - 3V_{l2\sigma} + (V_{l2\tau} - 3V_{l2\sigma\tau})(4T-3)] c_\alpha^{(1)2}(k_F r) \left. \left. (f_\alpha^{(1)})^2 \right] \right. \\ & + \sum_{k=2,3} [f_\alpha^{(k)'}{}^2 a_\alpha^{(k)2}(k_F r) + \frac{2m}{\hbar^2} (\{V_c + V_\sigma + (-6k+14)V_t - (k-1)V_{ls} \\ & + [V_\tau + V_{\sigma\tau} + (-6k+14)V_{t\tau} - (k-1)V_{ls\tau}](4T-3) - [V_T + V_{\sigma T} \\ & + (-6k+14)V_{tT}](4T)\} a_\alpha^{(k)2}(k_F r) + [V_{l2} + V_{l2\sigma} + (V_{l2\tau} + V_{l2\sigma\tau})(4T-3)] c_\alpha^{(k)2}(k_F r) \\ & + [(V_{ls2} + V_{ls2\tau})(4T-3)] d_\alpha^{(k)2}(k_F r) \left. \left. f_\alpha^{(k)2} \right] + \frac{2m}{\hbar^2} \{ [(V_{ls\tau} - 2(V_{l2\sigma\tau} + V_{l2\tau}) - 3V_{ls2\tau}) \right. \right. \\ & \times (4T-3)] + V_{ls} - 2(V_{l2} + V_{l2\sigma}) - 3V_{ls2} \} b_\alpha^2(k_F r) f_\alpha^{(2)} f_\alpha^{(3)} \\ & \left. \left. + \frac{1}{r^2} (f_\alpha^{(2)} - f_\alpha^{(3)})^2 b_\alpha^2(k_F r) \right\} \right\}, \quad (7) \end{aligned}$$

where $\alpha = \{J, L, T, S, S_z\}$ and the coefficient $a_\alpha^{(1)2}$, etc., are defined as

$$a_\alpha^{(1)2}(x) = x^2 I_{L, S_z}(x), \quad (8)$$

$$a_\alpha^{(2)2}(x) = x^2 [\beta I_{J-1, S_z}(x) + \gamma I_{J+1, S_z}(x)], \quad (9)$$

$$a_\alpha^{(3)2}(x) = x^2 [\gamma I_{J-1, S_z}(x) + \beta I_{J+1, S_z}(x)], \quad (10)$$

$$b_\alpha^{(2)}(x) = x^2 [\beta_{23} I_{J-1, S_z}(x) - \beta_{23} I_{J+1, S_z}(x)], \quad (11)$$

$$c_\alpha^{(1)2}(x) = x^2 \nu_1 I_{L, S_z}(x), \quad (12)$$

$$c_\alpha^{(2)2}(x) = x^2 [\eta_2 I_{J-1, S_z}(x) + \nu_2 I_{J+1, S_z}(x)], \quad (13)$$

$$c_\alpha^{(3)^2}(x) = x^2[\eta_3 I_{J-1, S_z}(x) + \nu_3 I_{J+1, S_z}(x)], \quad (14)$$

$$d_\alpha^{(2)^2}(x) = x^2[\xi_2 I_{J-1, S_z}(x) + \lambda_2 I_{J+1, S_z}(x)], \quad (15)$$

$$d_\alpha^{(3)^2}(x) = x^2[\xi_3 I_{J-1, S_z}(x) + \lambda_3 I_{J+1, S_z}(x)], \quad (16)$$

with

$$\beta = \frac{J+1}{2J+1}, \quad \gamma = \frac{J}{2J+1}, \quad \beta_{23} = \frac{2J(J+1)}{2J+1}, \quad (17)$$

$$\nu_1 = L(L+1), \quad \nu_2 = \frac{J^2(J+1)}{2J+1}, \quad \nu_3 = \frac{J^3 + 2J^2 + 3J + 2}{2J+1}, \quad (18)$$

$$\eta_2 = \frac{J(J^2 + 2J + 1)}{2J+1}, \quad \eta_3 = \frac{J(J^2 + J + 2)}{2J+1}, \quad (19)$$

$$\xi_2 = \frac{J^3 + 2J^2 + 2J + 1}{2J+1}, \quad \xi_3 = \frac{J(J^2 + J + 4)}{2J+1}, \quad (20)$$

$$\lambda_2 = \frac{J(J^2 + J + 1)}{2J+1}, \quad \lambda_3 = \frac{J^3 + 2J^2 + 5J + 4}{2J+1}, \quad (21)$$

and

$$I_{J, S_z}(x) = \int dq \, q^2 P_{S_z}(q) J_J^2(xq). \quad (22)$$

$J_J(x)$ is the familiar Bessel function and $P_{S_z}(q)$ is defined as

$$\begin{aligned} P_{S_z}(q) = & \frac{2}{3}\pi[(k_F^{\sigma_{z1}})^3 + (k_F^{\sigma_{z2}})^3 - \frac{3}{2}((k_F^{\sigma_{z1}})^2 + (k_F^{\sigma_{z2}})^2)q \\ & - \frac{3}{16}((k_F^{\sigma_{z1}})^2 - (k_F^{\sigma_{z2}})^2)^2 q^{-1} + q^3] \end{aligned} \quad (23)$$

for $\frac{1}{2}|k_F^{\sigma_{z1}} - k_F^{\sigma_{z2}}| < q < \frac{1}{2}|k_F^{\sigma_{z1}} + k_F^{\sigma_{z2}}|$,

$$P_{S_z}(q) = \frac{4}{3}\pi \min((k_F^{\sigma_{z1}})^3, (k_F^{\sigma_{z2}})^3) \quad (24)$$

for $q < \frac{1}{2}|k_F^{\sigma_{z1}} - k_F^{\sigma_{z2}}|$, and

$$P_{S_z}(q) = 0 \quad (25)$$

for $q > \frac{1}{2}|k_F^{\sigma_{z1}} + k_F^{\sigma_{z2}}|$, where $k_F^{(i)} = (3\pi^2 \rho^{(i)})^{\frac{1}{3}}$ and σ_{z1} or $\sigma_{z2} = +1, -1$ for spin-up and spin-down nucleons, respectively.

According to LOCV formalism, the two-body energy is minimized with respect to the variations in the functions $f_\alpha^{(i)}$ subject to the normalization constraint [27],

$$\frac{1}{A} \sum_{ij} \langle ij | h_{S_z}^2 - f^2(12) | ij \rangle_a = 0. \quad (26)$$

In the case of spin polarized nuclear matter, the function $h_{S_z}(r)$ is introduced as follows,

$$h_{S_z}(r) = \begin{cases} \left[1 - \frac{9}{2} \left(\frac{J_J^2(k_F^{(i)} r)}{k_F^{(i)} r} \right)^2 \right]^{-1/2} & ; \quad S_z = \pm 1 \\ 1 & ; \quad S_z = 0 \end{cases} \quad (27)$$

The minimization of the two-body cluster energy leads to a set of differential equations with the same form as those presented in Ref. [27], with coefficients replaced by Eqs. (8)–(16). The correlation functions, and then the two-body energy term are obtained through the solving of the differential equations.

Since, we consider the nuclear matter which is under the influence of a strong magnetic field (B), we must add the contribution of magnetic energy to the energy of system (Eq. (3)). By considering the magnetic field along the z axis, the contribution of the magnetic energy per nucleon is given by

$$E_M = -\frac{1}{A} \sum_i \vec{\mu}_i \cdot \vec{B} = \mu \xi B, \quad (28)$$

where μ is the magnetic dipole moment of nucleons.

III. RESULTS

Fig. 1 presents the dependence of the energy per particle on the spin polarization parameter at different magnetic fields and densities. Obviously, in the absence of a magnetic field, the energy per particle is a symmetric function of the spin polarization parameter. This shows that for the nuclear matter, the spontaneous phase transition to a ferromagnetic state does not occur [15]. Obviously, for $B \gtrsim 10^{18}$ G, the spin polarization symmetry is broken and the energy has a minimum value at a positive spin polarization parameter. The stronger the magnetic field is, the more the symmetry is broken. The comparison between top and bottom panels shows that the effects of magnetic fields are more important at lower densities. The degree of symmetry breaking, as well as the difference of the energy in strong magnetic fields and zero fields, is clearly higher at lower densities. Fig. 1 also shows that for $B \gtrsim 10^{18}$ G, at each density and spin polarization parameter, the energy grows with increasing the magnetic field.

The equilibrium value for the spin polarization parameter varies with both magnetic field and density, as shown in Fig. 2. As we mentioned above, in the free field case, the spin

polarization parameter is equal to zero, showing that no spontaneous phase transition to a ferromagnetic state occurs. For the magnetic fields $B \lesssim 10^{17} \text{ G}$, the nuclear matter is nearly unpolarized, but at $B \gtrsim 10^{18} \text{ G}$, the spin polarization parameter grows as the magnetic field increases. This parameter is more significantly affected by the magnetic field at lower densities. For example, at $\rho = 0.05 \text{ fm}^{-3}$ and $B = 10^{19} \text{ G}$, the nuclear matter is polarized with $\xi \simeq 0.69$. For each value of the magnetic field, the spin polarization parameter decreases by increasing the density. This behavior has been also reported in Ref. [10]. Our results show that at $\rho = 0.5 \text{ fm}^{-3}$ and $B = 10^{19} \text{ G}$, the spin polarization parameter is less than 0.2.

The magnetic field dependence of the spin polarization parameter at different densities is presented in Fig. 3. We found that at each density, by increasing the magnetic field up to $B \simeq 10^{17} \text{ G}$, the nuclear spin polarization parameter is nearly constant. However, for the magnetic fields $B \gtrsim 3 \times 10^{17} \text{ G}$, this parameter grows by increasing the magnetic field. Evidently, as the density increases, the stronger magnetic fields are needed to affect the spin polarization parameter, consistent with the results obtained in Ref. [10]. The increasing rate of the spin polarization parameter versus the magnetic field depends on the nucleon density. At lower densities, this parameter is more sensitive to the magnetic field and grows more rapidly. Obviously, at each density, the increasing rate of the spin polarization parameter becomes significant at a particular value of the magnetic field. For example at $\rho = 0.05 \text{ fm}^{-3}$, the increasing rate of ξ raises at $B \simeq 3 \times 10^{18} \text{ G}$. This value of the magnetic field increases by increasing the density.

The energy for nuclear matter as a function of the density at different magnetic fields is plotted in Fig. 4. We can see that for $B \gtrsim 10^{18} \text{ G}$, at each density, the energy of the system increases as the magnetic field grows. Evidently, the effect of strong magnetic fields is more considerable at lower densities. Fig. 4 implies that for all magnetic fields studied in this work, the nuclear matter becomes bound at a particular value of the density. The magnetic field dependence of the nuclear matter energy at different densities is shown in Fig. 5. We can see that at high magnetic fields, the nuclear matter energy increases as the magnetic field grows. The magnetic field necessary to affect the nuclear matter energy depends on the nucleon density. The higher the density becomes, the stronger the magnetic field necessary is. In fact, the magnetic effects on the nuclear matter are significant at low densities and high magnetic fields. Thus, to affect the high density nuclear matter, very strong magnetic

field is needed. Fig. 5 also implies that for high magnetic fields, the increase in the nuclear matter energy is more important at lower densities.

IV. SUMMARY AND CONCLUDING REMARKS

Using LOCV method and the AV_{18} two-body potential, we have considered the effects of strong magnetic fields on the properties of nuclear matter. It has been shown that for the nuclear matter, the spontaneous phase transition to a ferromagnetic state does not occur, while the spin polarization symmetry of energy is broken at high magnetic fields. We have concluded that the spin polarization parameter decreases as the density increases, while it grows with increasing the magnetic field. Our results show that the energy of the system increases as the magnetic field grows. Moreover, It has been found that the magnetic effects on the nuclear matter are significant at low densities and high magnetic fields.

Acknowledgments

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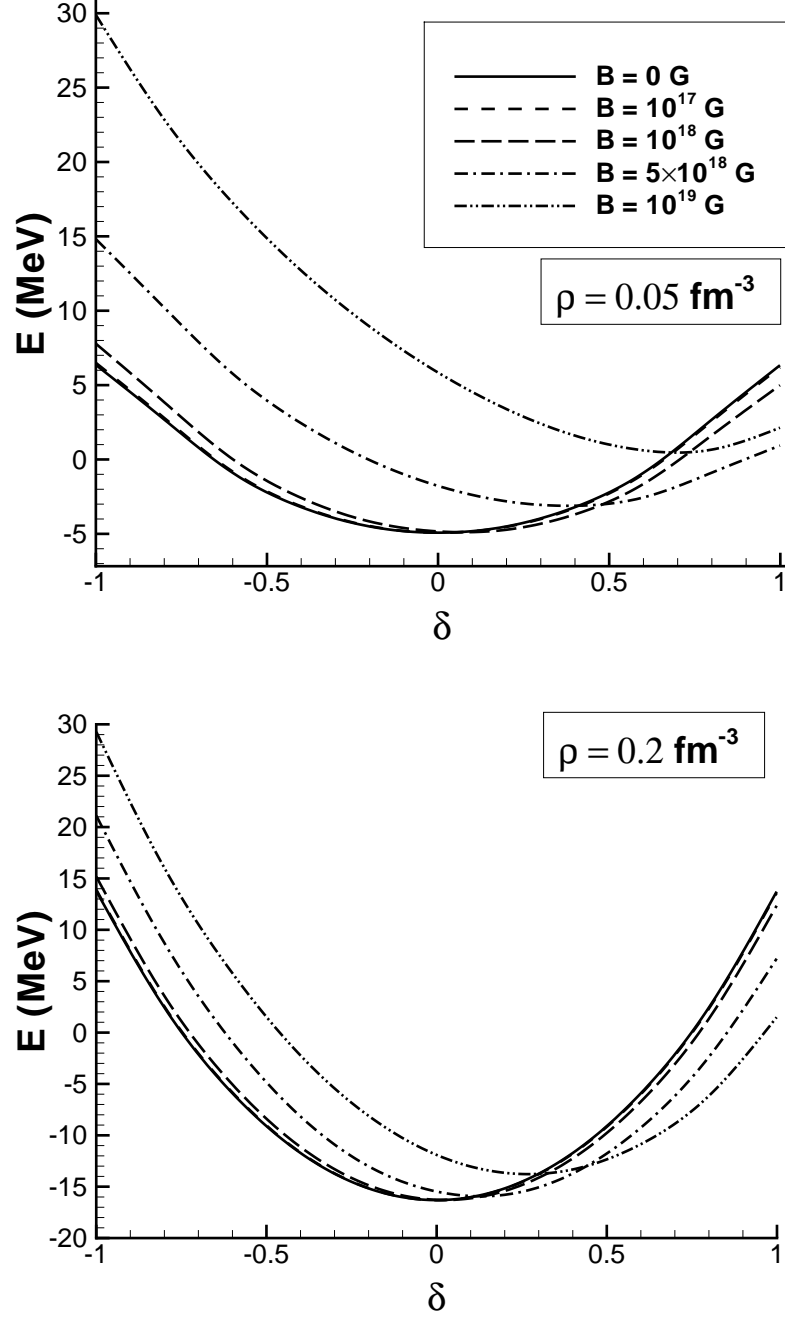


FIG. 1: The energy per particle versus the spin polarization parameter for the cases $B = 0 \text{ G}$ (solid curve), $B = 10^{17} \text{ G}$ (dashed curve), $B = 10^{18} \text{ G}$ (long dashed curve), $B = 5 \times 10^{18} \text{ G}$ (dashdot curve) and $B = 10^{19} \text{ G}$ (dashdotdot curve), and two values of the density, $\rho = 0.05 \text{ fm}^{-3}$ (top panel) and $\rho = 0.2 \text{ fm}^{-3}$ (bottom panel).

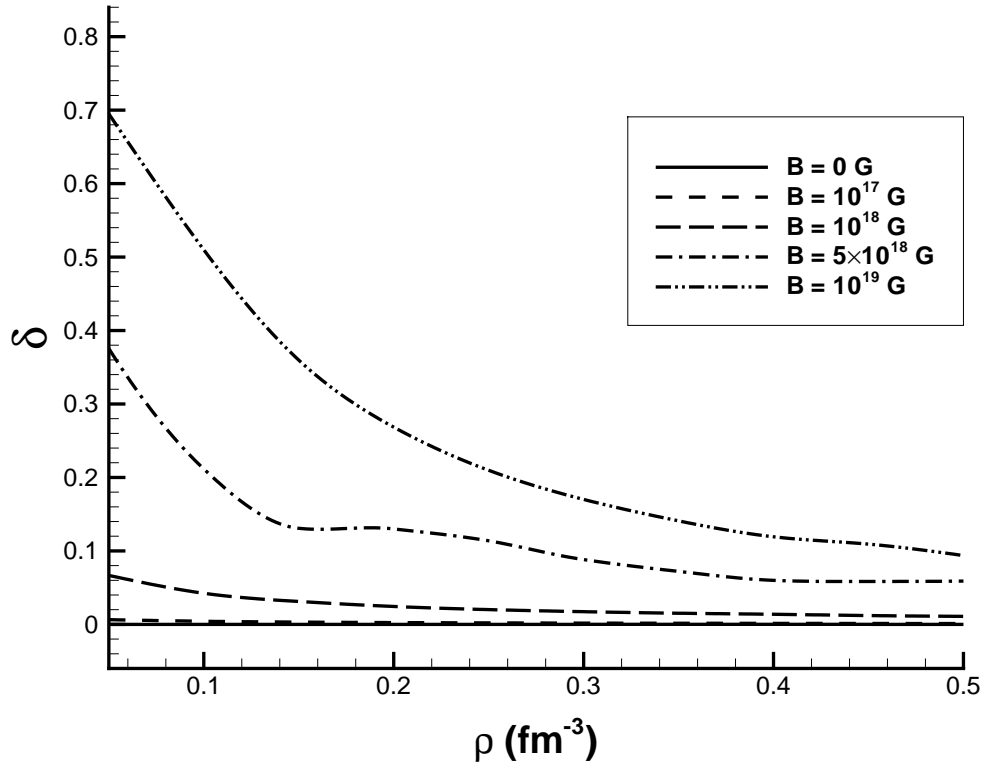


FIG. 2: Spin polarization parameter at the equilibrium state as a function of the density for the cases $B = 0 \text{ G}$ (solid curve), $B = 10^{17} \text{ G}$ (dashed curve), $B = 10^{18} \text{ G}$ (long dashed curve), $B = 5 \times 10^{18} \text{ G}$ (dashdot curve) and $B = 10^{19} \text{ G}$ (dashdotdot curve).

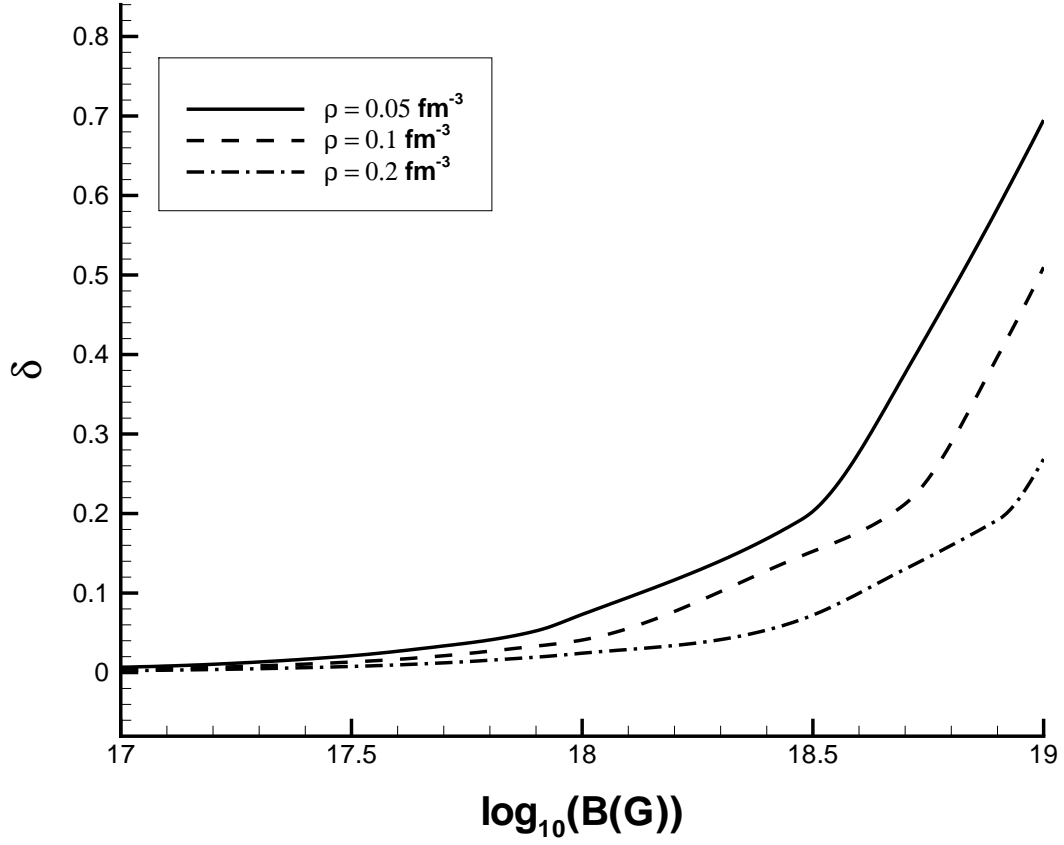


FIG. 3: Spin polarization parameter at the equilibrium state as a function of the magnetic field for the cases $\rho = 0.05 \text{ fm}^{-3}$ (solid curve), $\rho = 0.1 \text{ fm}^{-3}$ (dashed curve) and $\rho = 0.2 \text{ fm}^{-3}$ (dashdot curve).

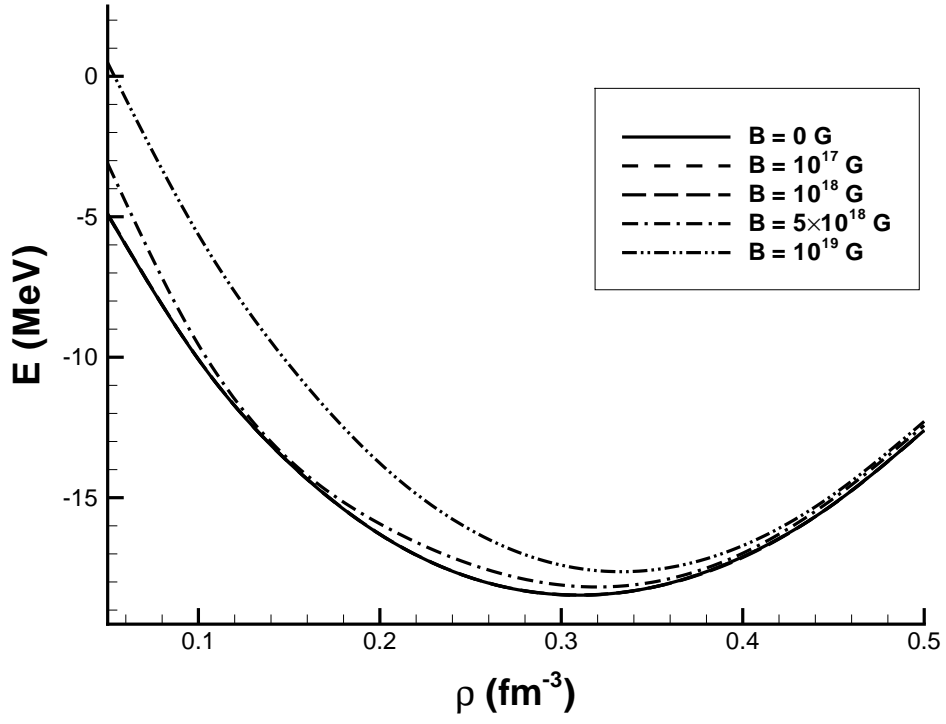


FIG. 4: The energy as a function of the density for the cases $B = 0 \text{ G}$ (solid curve), $B = 10^{17} \text{ G}$ (dashed curve), $B = 10^{18} \text{ G}$ (long dashed curve), $B = 5 \times 10^{18} \text{ G}$ (dashdot curve) and $B = 10^{19} \text{ G}$ (dashdotdot curve).

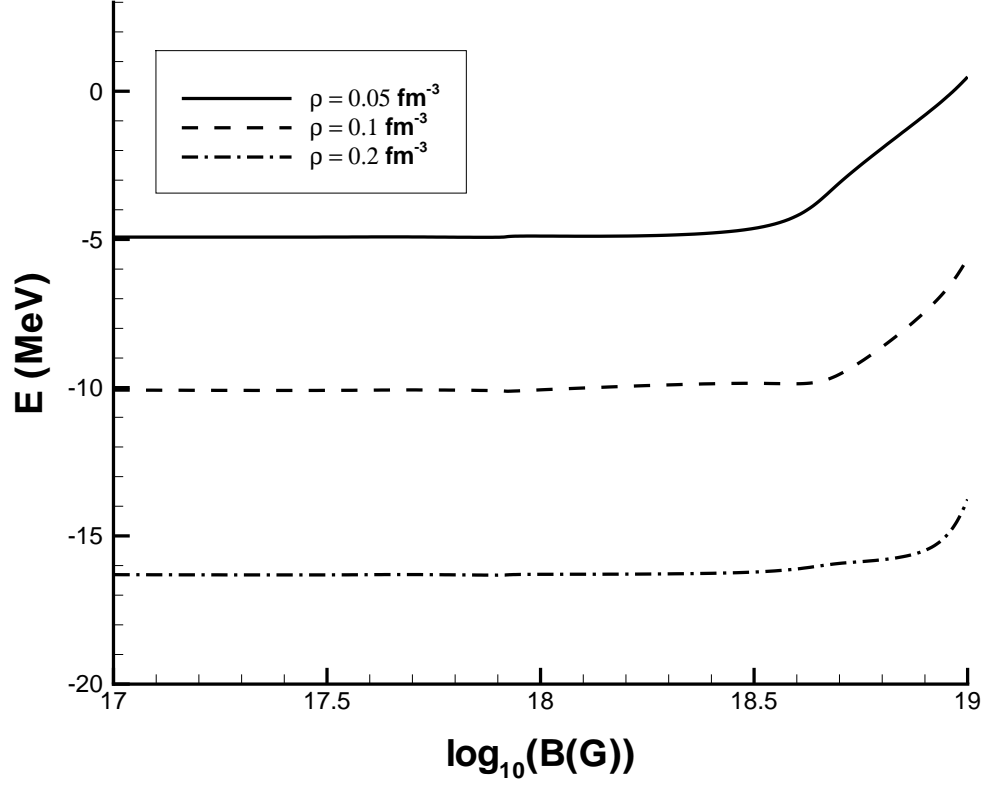


FIG. 5: The energy as a function of the magnetic field for the cases $\rho = 0.05 \text{ fm}^{-3}$ (solid curve), $\rho = 0.1 \text{ fm}^{-3}$ (dashed curve) and $\rho = 0.2 \text{ fm}^{-3}$ (dashdot curve).